Low-loss left-handed materials using metallic magnetic cylinders

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We discuss materials based on arrays of metallic magnetic cylindrical structures near ferromagnetic resonance with applied magnetic fields at microwave frequencies. We have found that the materials have a negative refraction index by combining the effective negative permittivity and permeability. Numerical finite-difference time-domain simulations were performed, after a very large number of geometries were swept. The simulations reveal that ferromagnetic cylinders, with diameters of 0.1 cm and 0.5 cm apart, and with periodic or random configurations, are left-handed materials with very small losses; i.e., with transmittivity practically unity or no loss. The way to obtain convenient structures and geometries is also discussed.

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Recently, there have been reports of man-made materials called metamaterials, formed by periodic arrays of cylinders and split ring resonators [1], which may exhibit a negative refractive index [2,3]. These materials, whose properties were theoretically predicted by Veselago [4], are known as left-handed media (LHMs). The materials that were reported so far, however, exhibited significant losses and a small transmission of the order of or smaller than 10^{-3}, meaning there is a significant problem with propagation in these systems [5]. Utilizing practically the same metamaterials, other experiments showed much larger transmission. In this case, however, the claimed permittivity and permeability had negative imaginary parts [6]. Regardless, the realization of these LHM materials is certainly interesting. It is, furthermore, convenient to obtain them in other ways with low losses, so that the propagation is not a problem, and with the real parts of the permittivity and permeability negative, while the imaginary parts are positive.

It is well known that at ferromagnetic resonance with right or linearly polarized radiation there is a frequency region where the real part of the permeability is negative and its imaginary part is positive with a small value [see Fig. 1(a)] [7]. In addition, for *s*-polarized radiation, an array of cylinders can exhibit a region where the same thing happens with the permittivity when the diameter and the separation between cylinders [see Fig. 1(b)] are adequately chosen. We, therefore, have the ingredients to fabricate LHM materials by arranging that the frequency regions of negative permittivity and permeability overlap. The only problem is whether it is possible to choose frequencies and geometrical structures that will allow low losses. Notice, for example, that a film of ferromagnetic metal will exhibit the real part of the refraction index as negative, but the losses at microwave frequencies will be very large. For a thickness of the order of the wavelength of the radiation ($\sim$1.5 cm), the transmittivity will be practically zero so that there is only attenuation of the radiation.

In this paper, we take the opportunity given by the properties of magnetic materials near ferromagnetic resonance (FMR) to elaborate on the above ideas, and discuss materials based on a structure consisting of metallic magnetic cylinders, although they can be generalized, under the influence of magnetic fields. A LHM has negative permittivity $\varepsilon(\omega)$, permeability $\mu(\omega)$, and refractive index $n(\omega)$, where ω is the frequency. The LHM is, moreover, a dispersive medium [4] with $\mathrm{Re}[n(\omega)]<0$ and an absorption that is determined by $\mathrm{Im}[n(\omega)]>0$. The propagation of the electromagnetic wave in LHMs exists only if $\mathrm{Im}[n(\omega)]$ is small enough, and the damping is less then $1/e$ in thickness (of the order of magnitude of or larger than the wavelength of the radiation).

We study the propagation in the frequency region where ferromagnetic resonance exists; i.e., $f<1-100$ GHz($\omega=2\pi f$). In the proximity of resonance μ changes sign, and becomes zero at a certain frequency ω_{cm}. For the region of frequencies between the resonance frequency ω_0 and the characteristic ω_{cm} we have $\mathrm{Re}[n(\omega)]<0$ and $\mathrm{Im}[n(\omega)]$ can be very small [see Fig. 1(a)]. A similar effect occurs for the dielectric behavior [5]. The metallic structure can be arranged (choosing separation of rods and diameter), as will be discussed below, to accommodate the negative values for permittivity and permeability in the same range of frequencies producing a LHM. Precisely, our calculations are oriented to show that there is a region of frequencies where the permittivity and permeability are both negative and the structure has LHM properties with low losses. The geometry that we have chosen here is not the only one but the extension to other structures is also discussed.

We consider an array of magnetic metallic wires embedded in a dielectric medium. In our case, this is air to prevent losses due to the dielectric. We do not need a medium to hold the wires because they can be held from their ends, like the cords of a guitar for example. We then simulate the electromagnetic wave propagation in a structure consisting of periodic or random rows of metallic cylinders. The metallic cylinders were assumed according with physics to have a frequency dependent dielectric permittivity

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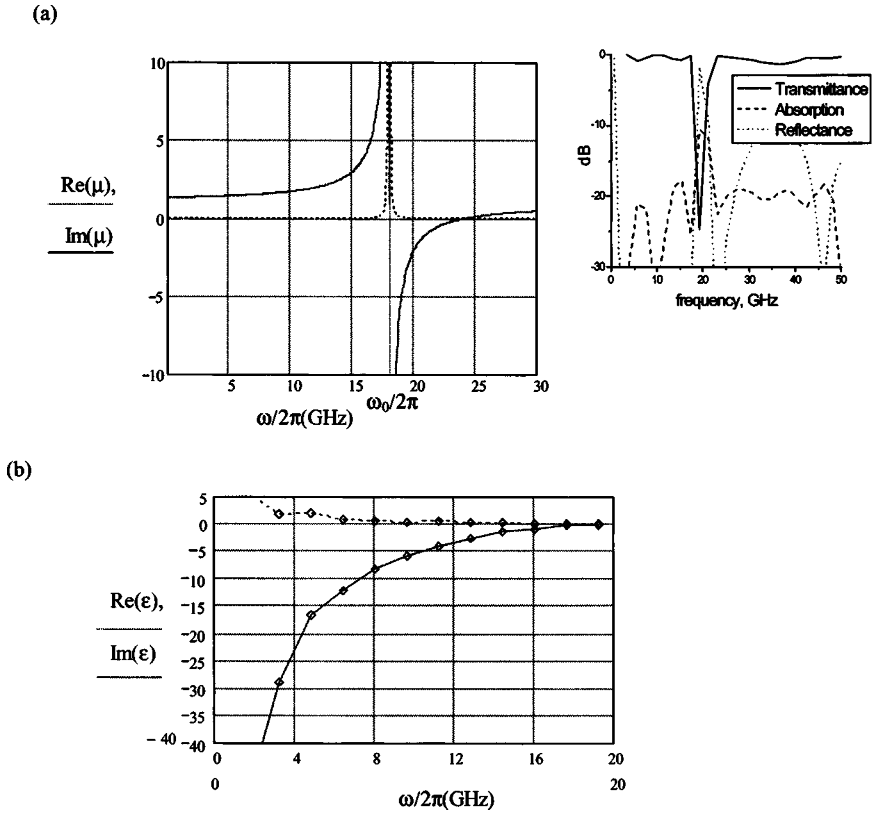

FIG. 1. (a) The magnetic permeability of a ferromagnetic material with $\omega_M/2\pi = 6$ GHz, $\alpha = 0.001$, and $\omega_0/2\pi = 18$ GHz. The inset shows the transmittance for the structure of the cylinders with $\varepsilon = 1$ and the permeability μ given by Eq. (3). (b) The effective dielectric permittivity of a Ni cylinder structure with diameter $d = 0.1$ cm and distance between the cylinders $a = 0.5$ cm.

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}, \tag{1}$$

where $f_p = 9$ eV is the plasmon frequency, and $\gamma = 0.1$ eV is the damping frequency taken to fit the experimental values of the permittivity [8]. Notice that in the range of microwave frequencies, for metals, typical values are $\varepsilon(\omega) \approx -5000i + 10^7$, which is general for all good metals. The transmission of the considered structure should have a cutoff frequency at about

$$f_c = 2c/(a - d), \tag{2}$$

where c is the velocity of light, a the separation between the cylinders, and d their diameter [5]. To the left of this frequency (smaller frequencies) the effective dielectric constant that takes into account the geometry is negative and it is positive to the right [see Fig. 1(b)].

The permeability, however, has a resonant behavior for ferromagnetic wires at the microwave frequency range discussed above. The frequency dependent permeability at ferromagnetic resonance for right-hand and linear radiation is

$$\mu(\omega) = 1 - \frac{\omega_M(\omega_0 + \omega - i\alpha\omega)}{\omega^2 - (\omega_0 - i\alpha\omega)^2}, \tag{3}$$

where ω_M is the effective characteristic frequency of the ferromagnet at magnetization M, ω_0 is the resonant frequency, γ_M being the gyromagnetic ratio, and α representing the friction coefficient [7]. The resonant frequency ω_0 depends on the demagnetizing field [7,9]. In addition, for fields perpendicular and parallel to the cylinders, ω_0 takes the values $\gamma_M[H(H-2\pi M)]^{1/2}$ and $\gamma_M(H+2\pi M)$, where H is the applied magnetic field. By changing the magnitude and direction of the field we can locate the resonance in a wide range of frequencies. This is favorable because it gives a lot of versatility for the frequency in which we want to have the LHM with the accompanying negative refraction. This calculation solves numerically the equations for the scalar approximation for permittivity and permeability. In this way it can be considered that within the model the calculation is exact. However, an interesting and maybe relevant point is that these physical parameters are tensors. The nonconsidered off diagonal part of the tensors can be important and change the results. However the tensorial model is out of the range of our possibilities and perhaps out of the range of any other calculation at present. Therefore our calculation is at the state of the art.

In Fig. 1(a) one can see the real and imaginary parts of the permeability vs frequency $f = \omega/2\pi$. The imaginary part has a large peak at the frequency ω_0, and the real part is negative in the region of frequencies between ω_0 and the characteristic frequency [approximately around $\omega_{cm} \approx (\omega_M + \omega_0)$]. The value of the imaginary part, furthermore, depends on the friction coefficient α. As an example [Fig. 1(a)], the permeability for $\omega_M/2\pi = 6$ GHz, $\omega_0/2\pi = 18$ GHz, and $\alpha = 0.001$ is shown. Near the frequency $\omega_{cm,}$ Im(μ) is small enough for reasonable values of the friction coefficient ($0.001 < \alpha < 0.01$) that yield typical resonance widths of 0.09–0.4 GHz [7,9]. Thus, in the region where $\mu \leqslant 0$ and ε has a negative value we could have very low losses and a pronounced peak of transmission. In fact if the imaginary parts are small enough we will have a good LHM. The question remains in finding the parameters that reduce losses. For that we perform extensive calculations.

For our simulation we used a frequency dependent finite-difference time-domain (FDTD) method [5,10]. We model the incident wave as an *s*-polarized pulse. The periodic boundary conditions were used along the wave propagation direction, and the absorption boundary conditions were used in the direction perpendicular to the wave propagation. The calculations were done with grid $dx = dy = a/50 = 0.01$ cm and time step $dt = 5.6 \times 10^{-4} a/c$, where c is the light velocity. Satisfactory convergence tests have been carried out calculating the propagation for different grid spacing. Now we have to simulate our model for the intercylinder distance and the best cylinder diameter. We choose $a = 0.5$ cm, which gives the cutoff frequency and scale at the values we want for the LHM region. We have chosen this value because through formula (2) we see that the cutoff is the region in which other LHMs have been tried and concord with the characteristic FMR frequencies. But other values of a can be chosen if the FMR resonance is in another frequency range. Notice that by increasing or reducing a in formula (2) the frequency of the cutoff is reduced or increased. The value d of the cylinder diameter was explored extensively between 0.001 and 0.2 cm. This variable is the key ingredient in the modeling, and the value chosen in the frequency region of interest (18–20 GHz) was the one resulting in small losses and a greater degree of transmission [11]; i.e., $d = 0.1$ cm. Other values explored give also $\mathrm{Re}[n(\omega)] < 0$ but the losses are larger. The structure has an effective negative permittivity with a low value for the imaginary part in a relative wide range of frequencies [Fig. 1(b)]. To calculate the effective ε we studied a metallic structure with the dielectric permittivity described by Eq. (1) and permeability μ equal to 1 everywhere, i.e., out of the FMR condition. As one can see from Fig. 1(b), the transmission has a cutoff frequency of about 25 GHz, and in the region below, the effective dielectric permittivity is negative and its imaginary part is negligible. So by optimizing the rod diameter we reduce losses.

We subsequently studied the transmission of the structure with ferromagnetic resonance where permeability inside the wires is taken as frequency dependent according to Eq. (3). Notice that now the calculations use the permittivity and permeability of the wires and solve exactly the Maxwell's equations for the structure. The values used for the parameters have been explored in a wide range and the best transmittivity was obtained when the reflectivity was practically zero because absorption is always small; this is guaranteed for the frequency region of interest. When the refractive index is close to −1 as is the case for $\omega_M/2\pi = 23$ GHz, $\omega_0/2\pi = 18$ GHz, and $\alpha = 0.001$ we obtain maximum transmission because there is no reflectivity (index of refraction near unity). For three rows of the cylinders, the results are shown in Fig. 2(a) for a periodic structure of the aforementioned structures for the normal incident wave. We have a big transmission in the small region around 19 GHz, a passband. This is where the permittivity ε and permeability μ are both negative, while the losses that are determined by the imaginary parts of ε and μ are small. To check the result further we have calculated also the effective permeability and this is negative in the region of frequencies from 18 to 21 GHz. The transmittance of the structure of the cylinders with $\varepsilon = 1$ and the permeability μ given by Eq. (3) is shown in the inset of Fig. 1(a). One can see a clear gap in the region around 19 Hz. The effective permeability for 19 GHz was calculated and it is $(-0.3 + 0.13i)$. Thus, we have a left-handed material with small losses in this area, and a transmission of practically

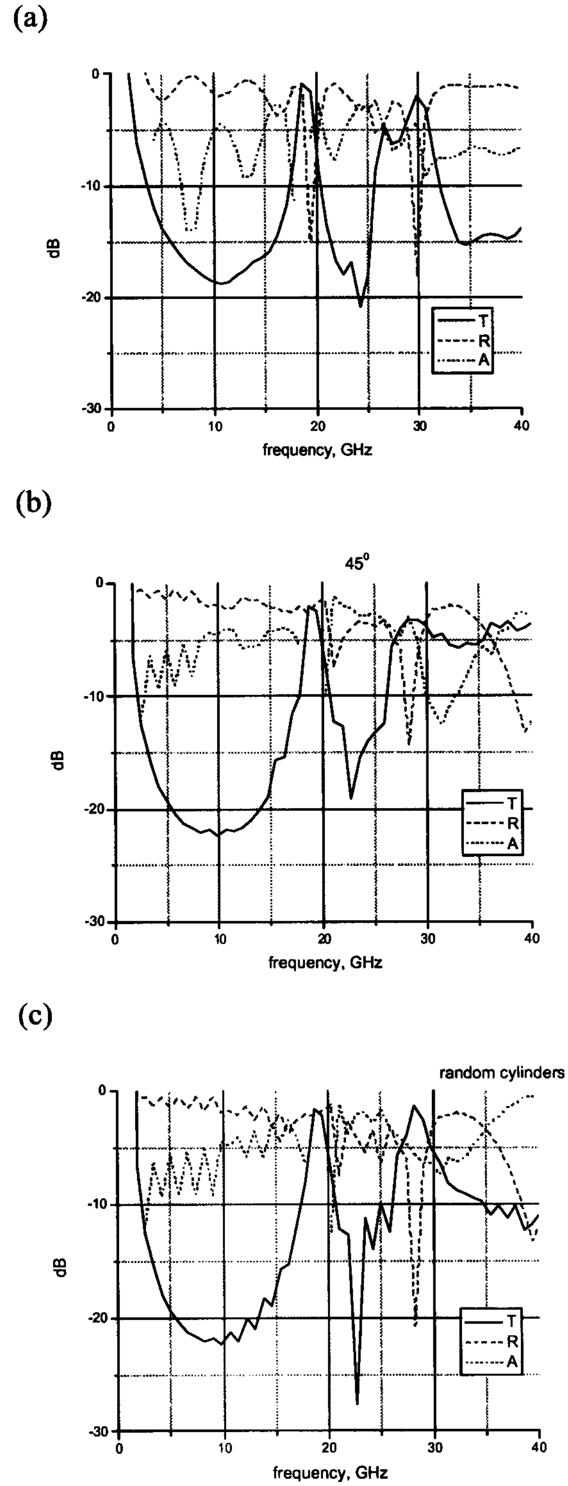

FIG. 2. The transmission, reflection, and absorption of a ferromagnetic cylinder structure with $d = 0.1$ cm and $a = 0.5$ cm for $\omega_M/2\pi = 23$ GHz, $\alpha = 0.001$, and $\omega_0/2\pi = 18$ GHz for (a) the normal incident wave, (b) 45° angle of incidence, and (c) a random cylinder structure for a normally incident wave.

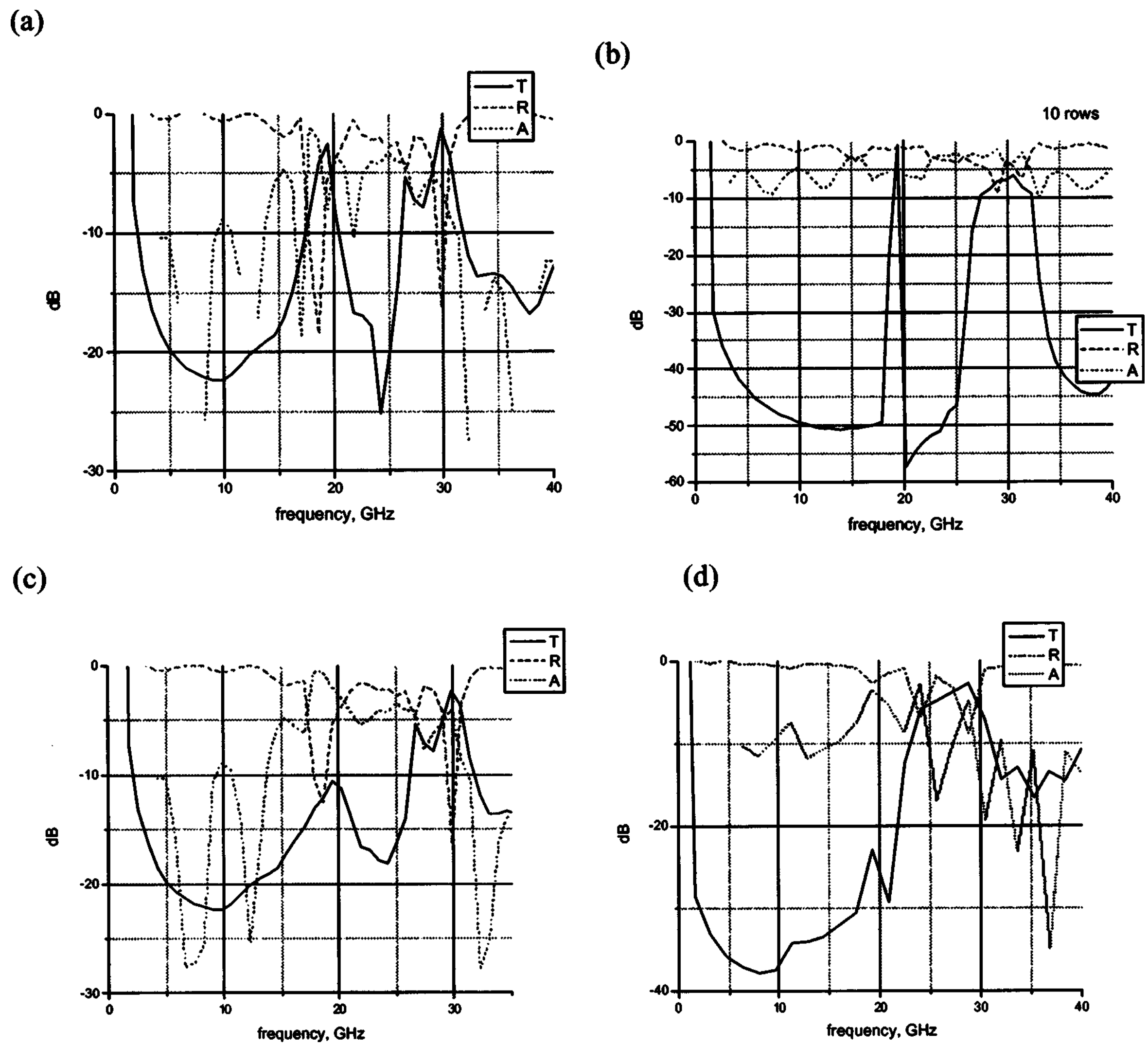

FIG. 3. The transmission, reflection, and absorption of a ferromagnetic cylinder structure with $d=0.1$ cm and $a=0.5$ cm. (a) and (b) $\omega_M/2\pi=23$ GHz, $\omega_0/2\pi=18$ GHz, and (a) three rows of cylinders with $\alpha=0.005$, (b) ten rows of cylinders with $\alpha=0.001$. (c) and (d) Three rows of cylinders, $\omega_0/2\pi=18$ GHz, $\alpha=0.04$, and $\omega_M/2\pi=$ (c) 23 and (d) 6 GHz.

unity (0 dB). The simulations were also done for an angle of incidence equal to 45° for the periodic structure [Fig. 2(b)], and for random cylinders with an average distance of $a=0.5$ cm with a normally incident wave [Fig. 2(c)]. The same behavior was observed in these cases as well. It shows the isotropic behavior of the material; no matter what the incident angle is, the peak appears at the same point: this is a key point because it permits us to define a refractive index and needs to be checked to guarantee the definition of the refractive index. All the results are similar and show a very high transmission near ω_0 where the both μ and ε are negative. This also gives versatility in the sense that we do not need a periodic structure. That is to say, hanging wires of ferromagnetic material, placed randomly with an average separation of 0.5 cm will yield LHM properties [Fig. 2(c)].

Increase of the friction coefficient α decreases the transmission a little. The region with high transmission can be, moreover, controlled by the value of the applied magnetic field H which determines the value of ω_0. In Fig. 3(a) the transmission for $\alpha=0.005$ is shown. One can see that it is 1.5 dB less than for $\alpha=0.001$. In order to prove that we have good propagation we have calculated the transmission of the structure while increasing the number of rows up to 10. We find that the peak narrows down, but has a transmittivity of unity for a well-determined frequency around 19 GHz [Fig. 3(b)]. The frequency ω_0 determines the position of the transmission peak. For p polarization there is also a large transmittivity around 19 GHz but the cutoff in permittivity is not so clear. However, the increase of α to the value $\alpha=0.04$ had the result of a big transmission decrease. In Fig. 3(c) one can see that the transmission is less than -10 dB in the peak around 19 GHz. With lower values of ω_M the transmission decreases even more and the peak almost disappears then ω_M is less than 6 GHz [Fig. 3(d)].

We made the same calculations for ω_0 equal to 15 and 10 GHz (Fig. 4). As a result, we had a corresponding transmission peak correspondingly near 15 or 10 GHz, though its value was a little smaller for ω_0 equal to 15 and significantly smaller for 10 GHz. The reduction of the peak of transmittivity occurs because the dielectric permittivity has a greater

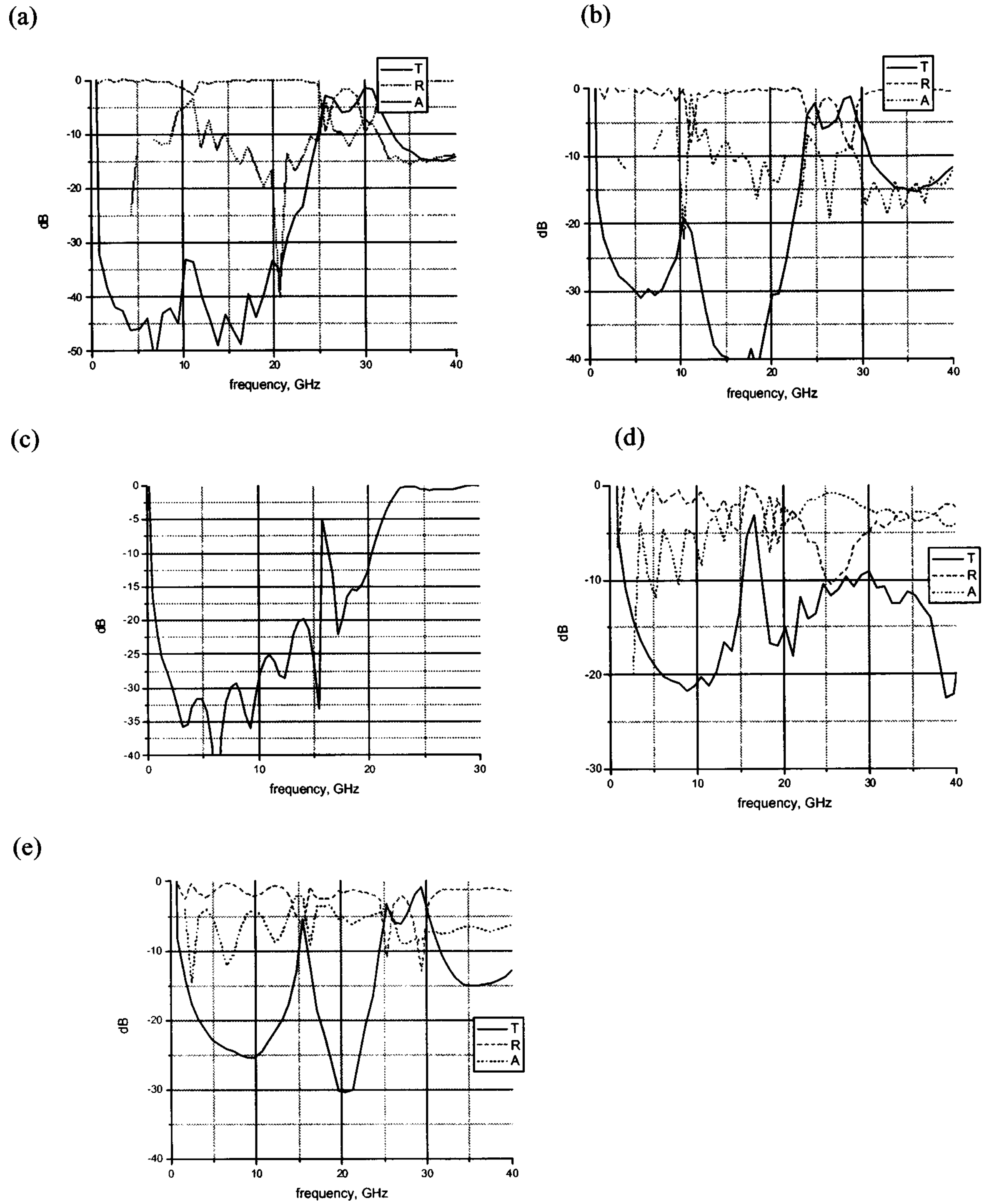

FIG. 4. The transmission, reflection, and absorption coefficients of a ferromagnetic cylinder with $d=0.1$ cm and $a=0.5$ cm. (a) and (b) $\omega_0/2\pi=10$ GHz for $\alpha=0.001$, and $\omega_M/2\pi=$ (a) 12 and (b) 23 GHz. (c), (d), and (e) $\omega_0/2\pi=15$ GHz for $\alpha=0.001$, and (c) $\omega_M/2\pi=24$ GHz, (d) as in (c) for 45° angle of incidence, and (d) $\omega_M/2\pi=20$ GHz.

negative value while its imaginary part is greater as well [see Fig. 1(b)]. The losses are exponentially dependent on the value $\mathrm{Im}(\varepsilon)\mathrm{Re}(\mu)+\mathrm{Im}(\mu)\mathrm{Re}(\varepsilon)$, and these increase as ω_0 is shifted toward the cutoff frequency. However, high transmittivity values can be obtained by decreasing the cutoff frequency of the structure; that is to say, by increasing the intercylinder distance a according to formula (2). In other words, we can simulate the LHM in the desired frequency region by changing the field H and the separation between cylinders. Transmission in the peak reduces with reduction of $\omega_M/2\pi$; this is seen on comparing Figs. 4(a) and 4(b) for $\omega_0/2\pi=10$ GHz and Figs. 4(c) and 4(d) for $\omega_0/2\pi=15$ GHz. By reducing the value of $\omega_M/2\pi$ to 6 GHz we obtain a transmittivity of 0.1 and a reflectivity of 0.9, with practically no absorption. The refractive index$=-0.06$ in this case [11] which is the reason for the calculated reflectivity

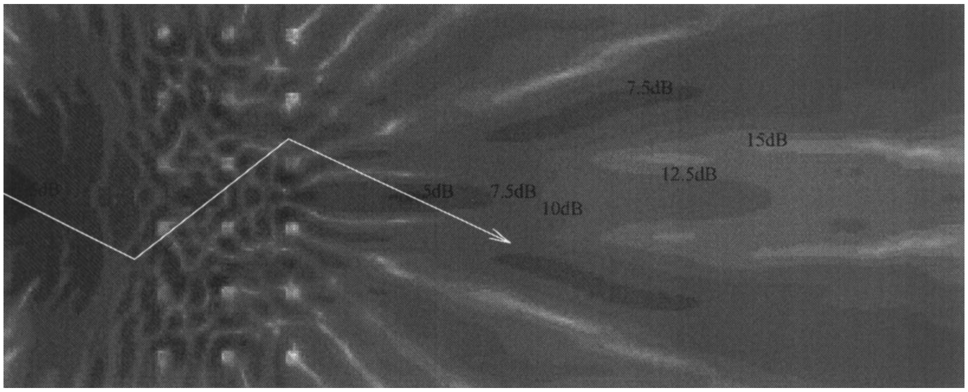

FIG. 5. The focusing properties of the material (3 rows of cylinders) for a point source located at 1 cm from the material. Notice the focus at 0.65 cm corresponding to an effective real part of the refraction index of −1.1 with a very small imaginary part given the large transmittivity. The three rows of cylinders are the very white dots (zero field) of the figure.

but the material is also a good LHM with low losses, although highly reflecting.

One more test is that a LHM should present focusing properties as was previously discussed [4]. We have simulated the transmittivity and focusing properties of a point source located at 1 cm from our material, at 19 GHz. Figure 5 shows the results for $\omega_M/2\pi=23$ GHz of characteristic magnetization with a clear focus at a 0.65 cm distance that corresponds to a refractive index of $n=-1.1$, with a very small imaginary part because the transmittivity is near unity and the reflectivity practically zero.

Therefore we can claim that good LHMs (low losses) can be obtained by following the instructions. (i) Define a magnetic material which should have low losses and have a characteristic magnetization. This together with the applied field defines a characteristic region where $\text{Re}(\mu)<0$ and $\text{Im}(\mu)$ is small because of the material elaboration. (ii) Chose the adequate intercylinder distance a to set the cutoff around the region of $\text{Re}(\mu)=<0(\omega_0)$. (iii) Optimize the value of d to have low losses in wires due to reducing $\text{Im}(\varepsilon)$. These three ingredients will provide a good, low-loss LHM. Naturally the possible values of FMR frequencies rule the region where this can be applied. For the permittivity there is a wider range of frequencies due to the range of values permitted by formula (2).

Another interesting aspect from our simulations, in the wide region of parameters used, is that there is a good region for LHMs with wires of approximately 0.001 cm in diameter. In this region, however, our calculations are not too precise due to the large ratio between the wavelength of the radiation and the wire diameter [11]. In addition, similar results can be obtained using spheres instead of wires, but this case is considerably more difficult to construct in order to have them hanging in air to reduce losses in the medium and to perform the simulations [11]. They can be fabricated also with a medium different from air, and thus allow another parameter into the manipulations to prepare the material. This may also be positive since there is large experience in composite materials. In any case a warning has to be given and this is that one needs a specialist in producing clean, magnetic domain walls structures, and low-loss magnetic wires adequate for FMR. This is a key point in obtaining real clean LHMs with low losses, not reached until now.

In conclusion, we have studied and calculated exactly the transmission properties of composite metallic media according to the behavior of their permittivity and permeability within the energy dependent scalar approximation (neglecting off diagonal tensorial components). We have shown the existence of interesting properties at microwave frequencies in the FMR region where $\text{Re}[\mu(\omega)]<0$, and where the medium has real values of negative n, in our case −1.1 as obtained from focusing properties. This allows us to obtain appreciable values of the transmission confined to a small range of frequencies. Our simulation shows that the structure of cylinders made from a ferromagnetic material can have left-handed properties under the action of a magnetic field H with low losses. A rule for accommodating the material parameters for different frequencies accessible to FRM is given by controlling the applied magnetic field and the distance between cylinders. Also other geometries have been discussed. We have shown that the losses of the material (α parameter) can destroy all the propagating properties. Therefore the losses of the material forming the wires will determine the properties of the composite and we have scanned different losses that can describe a range from single crystals to polycrystalline materials. This should be a very important parameter when setting an experiment.

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